THEORY OF PERFECT PLASTICITY OF COMPOSITE MATERIALS,

TAKING ACCOUNT OF VOLUME COMPRESSIBILITY

L. A. Saraev

UDC 539.4

The practical importance and timeliness of the study of the mechanical macroscopic behavior of composite microinhomogeneous media are determined by its ability to give criteria for estimating the limiting load of various structural elements, the flow of multiphase dispersed systems, the deformation of materials made by powder metallurgy methods, etc. The theoretical prediction of the properties of composites is generally most effectively realized when their structural representations are based on the theory of random fields. Application of the "strong isotropy" hypothesis [1] for the statistical averaging of certain relations of a perfectly plastic body with microstructure permits the determination of its macroscopic yield surface.

Suppose the mechanical properties of the components of a two-phase isotropic rigid-plastic body are described by a yield surface taking account of hydrostatic pressure

$$s_{ij}s_{ij} + a_\alpha \sigma_{ll} = k_\alpha^2, \quad \alpha = 1;\ 2,$$

where $s_{ij} = \sigma_{ij} - (1/3)\delta_{ij}\sigma_{ll}$; σ_{ij} is the stress tensor, the k_α are the plasticity limits of the phases, the a_α are parameters characterizing the volume compressibility of the components, and σ_{ll} is the first invariant of the stress tensor.

The relative position of the components in space, connected with one another by perfect adhesion, is such that the rate of displacement field $v_i(\mathbf{x})$ is continuous, is characterized by random isotropy of the indicator function $\varkappa(\mathbf{x})$, equal to unity at points of the first phase, and zero in the second. With its help the local associated yield law of the medium can be written in the form

$$\sigma_{ij}(\mathbf{x}) = k(\mathbf{x}) \frac{\varepsilon_{ij}(\mathbf{x}) - \delta_{ij} b(\mathbf{x}) \varepsilon_{ll}(\mathbf{x})}{\sqrt{\varepsilon_{mn}(\mathbf{x}) \varepsilon_{mn}(\mathbf{x}) - b(\mathbf{x}) \varepsilon_{ll}^2(\mathbf{x})}}. \tag{1}$$

Here $\varepsilon_{ij}(\mathbf{x})$ is the rate of strain tensor; $k(\mathbf{x}) = k_1 + [k]\varkappa(\mathbf{x})$; $b(\mathbf{x}) = b_1 + [b]\varkappa(\mathbf{x})$; $b_\alpha = (3a_\alpha + 1)/9$, square brackets denote values of the discontinuities of the parameters in passing through a phase boundary — $[f] = f_2 - f_1$. The function $\varkappa(\mathbf{x})$, the stresses, and the rates of strain are assumed to be random uniform and ergodic fields, and their mathematical expectations agree with the volume averages of the components V_α also over the total volume $V = V_1 + V_2$ [2]

$$\langle(\ldots)\rangle = \frac{1}{V}\int_V (\ldots)\,dV, \quad \langle(\ldots)\rangle_\alpha = \frac{1}{V_\alpha}\int_{V_\alpha} (\ldots)\,dV.$$

We neglect the fluctuations of the dissipative function in the volume V and write (1) in the form

$$\Lambda\sigma_{ij} = k^2(\mathbf{x})\,\varepsilon_{ij}(\mathbf{x}) - k^2(\mathbf{x})\,b(\mathbf{x})\,\delta_{ij}\varepsilon_{ll}^2, \tag{2}$$

where $\Lambda = \langle\sigma_{ij}\varepsilon_{ij}\rangle$. Substituting (2) into the equilibrium equations

$$\sigma_{ij,j} = 0,$$

we find

$$k_1^2\varepsilon'_{ij,j} - k_1^2 b_1 \varepsilon'_{ll,i} + F'_i = 0,$$

Kuibyshev. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 164-167, May-June, 1981. Original article submitted April 14, 1980.

0021-8944/81/2203-0435$07.50

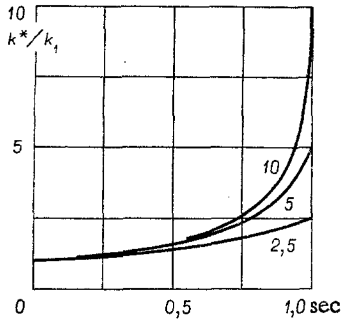

Fig. 1

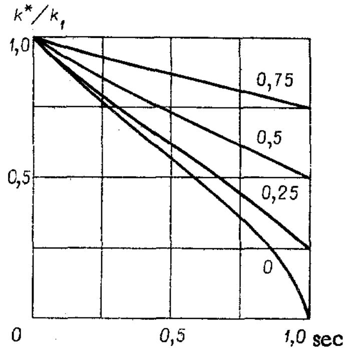

Fig. 2

where $F_i = [k^2](\varkappa\varepsilon_{ij})_{,j} - [k^2b](\varkappa\varepsilon_{ll})_{,i}$, and primes denote fluctuations of quantities. Adding the Cauchy relations

$$2\varepsilon'_{ij} = v'_{i,j} + v'_{j,i},$$

we obtain a closed system of equations for the $v_i'(\mathbf{x})$, which with the help of the Green tensor is transformed into a system of integral equations for the components of the $\varepsilon'_{ij}(\mathbf{x})$ tensor [3, 4]:

$$\varepsilon'_{ij}(\mathbf{x}) = \int_V G_{i(k,l)j}(\mathbf{x}-\boldsymbol{\xi})\cdot([k^2]\varkappa(\boldsymbol{\xi})\varepsilon_{kl}(\boldsymbol{\xi}) - \delta_{kl}[k^2b]\varkappa(\boldsymbol{\xi})\varepsilon_{mm}(\boldsymbol{\xi}) - \\ - [k^2]\langle\varkappa\varepsilon_{kl}\rangle + [k^2b]\delta_{kl}\langle\varkappa\varepsilon_{mm}\rangle)\,dV. \quad (3)$$

To determine the rheological relation between the volume averages of the stresses and rates of strain it is necessary to evaluate the tensor $\langle\varkappa'\varepsilon'_{ij}\rangle$. Since the composite under consideration is isotropic, the "strong isotropy" hypothesis can be applied. The correlation functions

$$\langle\varkappa'(\mathbf{x})\varkappa'(\boldsymbol{\xi})\varepsilon'_{kl}(\boldsymbol{\xi})\rangle$$

which appear under the integral sign in the multiplication of Eq. (3) by $\varkappa'(\mathbf{x})$ and the subsequent averaging over V are assumed to depend only on the distance $|\mathbf{x}-\boldsymbol{\xi}|$. Then [4]

$$\langle\varkappa'\varepsilon'_{ij}\rangle = \frac{c(1-c)}{15k_1^2(1+6a_1)}((6+54a_1)d_{ij} - \delta_{ij}(2+3a_1)d_{kk}).$$

Here $d_{ij} = -[k^2]\langle\varepsilon_{ij}\rangle_2 + \delta_{ij}[k^2b]\langle\varepsilon_{ll}\rangle_2$; $c = V_2V^{-1}$ is the volume fraction of the second phase. Adding the obvious relations

$$\langle\varepsilon_{ij}\rangle_2 = \langle\varepsilon_{ij}\rangle + c^{-1}\langle\varkappa'\varepsilon'_{ij}\rangle,$$

we obtain a system of equations for $\langle\varkappa'\varepsilon'_{ij}\rangle$. The substitution of the solution of this system into Eq. (2) averaged over V gives

$$\Lambda\langle\sigma_{ij}\rangle = A\langle\varepsilon_{ij}\rangle + \delta_{ij}\left(B - \frac{1}{3}A\right)\langle\varepsilon_{ll}\rangle, \quad (4)$$

where

$$A = k_1^2\left(1 + \frac{15c[k^2](1+6a_1)}{15k_1^2(1+6a_1) + 2(1-c)[k^2](3+27a_1)}\right);$$

$$B = \frac{k_1^2}{9}\,\frac{k_2^2 + 6k_1^2a_2 + 6c\left(k_2^2a_1 - k_1^2a_2\right)}{a_1\left(k_2^2 + 6k_1^2a_2\right) + c\left(k_1^2a_2 - k_2^2a_1\right)}.$$

We determine Λ from the relation

$$\int_V \sigma'_{ij}\varepsilon'_{ij}\,dV = 0,$$

obtained by multiplying the equilibrium equations by v_i' and integrating over the total volume of the composite. Now $\Lambda = \langle\sigma_{ij}\rangle\langle\varepsilon_{ij}\rangle$. Eliminating the tensor $\langle\varepsilon_{ij}\rangle$ from this and Eq. (4),

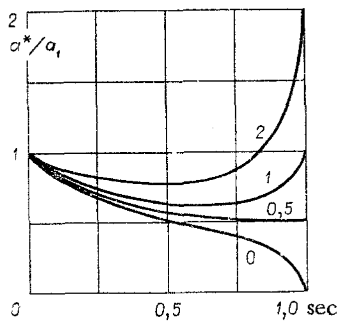

Fig. 3

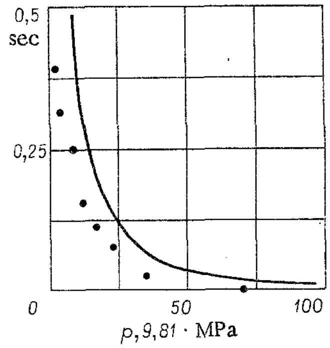

Fig. 4

we find the macroscopic yield surface of the microinhomogeneous medium under consideration

$$\langle s_{ij}\rangle\langle s_{ij}\rangle + a^*\langle\sigma_{ll}\rangle^2 = k^{*2}, \tag{5}$$

where $k^* = A^{1/2}$ is the effective plasticity limit of the composite, and $a^* = A/9B$ is its macroscopic parameter, taking account of volume compressibility.

Figures 1 and 2 show the dependence of the effective plasticity limit on the volume fraction of the second phase. The numbers on the curves are the values of the ratio k_2/k_1; $a_1 = 0.25$. The formula for k^* shows that the components of the composite take part in the deformation in different ways: if the second phase consists of voids and cavities ($k_2 = 0$), $k^* \not\equiv 0$; if the first phase is such voids ($k_1 = 0$), $k^* \equiv 0$. This means that the model constructed describes the properties of an inhomogeneous body in which the first component is the matrix and the second the inclusions.

Figure 3 shows the dependence of a^* on the volume fraction of the inclusions. The numbers on the curves are the values of the ratio a_2/a_1; $a_1 = 0.5$, $k_2/k_1 = 5$. It is interesting to note that for $a_1 = a_2$, $a^* \not\equiv 1$ and depends on the ratio of the plasticity limits. It can be seen from the expression for a^* that a composite material corresponding to the model constructed is macroscopically incompressible when and only when both parameters $a_1 = a_2 = 0$.

The proposed model of a microinhomogeneous rigid-plastic medium is graphically illustrated by a composite in which the inclusions are pores ($k_2 = 0$) and the material of the matrix is plastically incompressible ($a_1 = 0$). Then Eq. (5) takes the form

$$\langle s_{ij}\rangle\langle s_{ij}\rangle + \frac{c}{2(3+2c)}\langle\sigma_{ll}\rangle^2 = k_1^2\frac{3(1-c)}{3+2c}. \tag{6}$$

Equation (6) represents the macroscopic yield surface of an isotropic porous material whose matrix satisfies the Mises yield condition. Martynova and Skorokhod [5] investigated the mechanical properties of porous electrolytic nickel subjected to uniaxial compression in a mold. From the associated yield law corresponding to (6) it follows that the pressure p within such a mold is given by the expression

$$3p = \langle\sigma_{ll}\rangle = k_1\sqrt{\frac{2}{3}\,\frac{3-c-2c^2}{3c+5c^2}}.$$

The dependence of the porosity on the pressure is given by the expression

$$c = \left(\sqrt{(3m-1)^2+12(5m+2)} - 3m - 1\right)/(10m+4), \quad m = \frac{3p^2}{2k_1^2}.$$

Figure 4 compares the theoretical compaction curve (solid line) with the experimental points [5].

LITERATURE CITED

1. V. A. Lomakin, Statistical Problems of the Mechanics of Deformable Solids [in Russian], Nauka, Moscow (1970).
2. T. D. Shermergor, The Theory of Elasticity of Microinhomogeneous Media [in Russian], Nauka, Moscow (1977).

3. W. Nowacki, The Theory of Elasticity [Russian translation], Mir, Moscow (1975).
4. V. V. Dudukalenko, S. I. Meshkov, and L. A. Saraev, "Calculation of effective plasticity characteristics of inhomogeneous media," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1979).
5. I. F. Martynova and V. V. Skorokhod, "Compaction of porous metals under volume plastic deformation without strain hardening," Poroshk. Metall., No. 5 (1976).